List of errors in
and suggested modifications for
First-Order Modal Logic
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James W. Garson has answered a question we raised, in a paper that is of much interest for reasons far beyond this particular one. On page 185 we asked how to prove the converse Barcan formula in a system with the Barcan formula, axiom B, and no identity relation. In his paper "Unifying Quantified Modal Logic," he shows it cannot be done. His paper will appear in "The Journal of Philosophical Logic."

The following errors were discovered by members of a seminar, and communicated by one of its members, Thomas Bolander.

Chapter 5, Pages 127–128 The following fact is proved: If the systematically constructed tableau is infinite, then it must contain an infinite open branch. It might be a good idea to mention that this fact is a trivial consequence of König's Lemma and that the proof given is simply a proof of König's Lemma (in the special case where every node has at most two children). Many of the readers of the book might not know König's Lemma, but for those who do it is certainly helpful to have this mentioned.

Chapter 5, Page 129 One could mention that with respect to the Key Fact, only the first part is really needed for completeness. The second part, $\sigma \neg X$ on $\mathcal{B} \Longrightarrow \mathcal{M}, \sigma \not\Vdash_{v_0} X$, is only needed for the induction proof to go through.

Chapter 5, Page 129 In the proof of the Key Fact one is using the fact that any free variable in any formula on $\mathcal{B}$ is a parameter. One might mention that this follows from the fact that Φ is a closed formula (sentence) and from items 6 and 7 in the construction of the tableau.

Chapter 6, Page 138 Line 4 of proof: "Ex. 3.1.1" should be "Ex. 3.1.3 item 2".

Chapter 7, Pages 150–151 On page 117 one reads (as is the standard convention): "$\Phi(x)$ is a formula with some (possibly no) occurrences of the free variable x and $\Phi(y)$ is the result of replacing all free occurrences of x with occurrences of y." However, in Exercise 7.4.1, part 4 one reads "$\Phi(x)$ is a formula in which y does not occur, and $\Phi(y)$ is the result of substituting occurrences of y for free occurrences of x in $\Phi(x)$." It doesn't say "all occurrences" in the second formulation, so we were not quite sure whether you mean "all occurrences" or "some occurrences" at this place. The same kind of formulation is used in Definition 7.5.2.

Here is a reformulation, in which the notation is used as on page 117. Let $\Phi(u)$ be a formula in which no occurrence of u is within the scope of a quantifier binding either x or y. Then $(x = y) \supset (\Phi(x) \equiv \Phi(y))$.

Chapter 9, Page 198 In Definition 9.4.6 on reads "exactly as in Definition 4.6.7". This should be "exactly as in Definition 4.7.8". Otherwise only constant domain semantics are covered by Definition 9.4.6.

Chapter 10, Page 226 Exercise 10.6.4 asks you to give a varying domain tableau proof of a certain sentence. However, this sentence is not valid. The following is a countermodel:

$\mathcal{M} = \langle \mathcal{G}, \mathcal{R}, \mathcal{D}, \mathcal{I} \rangle$ where $\mathcal{G} = \{1, 2\}$, $\mathcal{R} = \{\langle 1, 2\rangle\}$, $\mathcal{D}(1) = \{a, b\}$, $\mathcal{I}(A, 1)$ is irrelevant. $\mathcal{I}(B, 1)$ is irrelevant, $\mathcal{I}(c, 1)$ is irrelevant, $\mathcal{I}(f, 1)$ is irrelevant, $\mathcal{D}(2) = \{a\}$, $\mathcal{I}(A, 2) = \{b\}$, $\mathcal{I}(B, 2) = \{b\}$, $\mathcal{I}(c, 2) = b$, $\mathcal{I}(f, 2) = \{\langle a, a\rangle, \langle b, a\rangle\}$.

The formula given in the exercise is false at world 1 of $\mathcal{M}$.

Chapter 11, Page 234 Lines 9–10: $(v * \mathcal{I})(t)$ should be $(v * \mathcal{I})(t, \Gamma)$, two occurrences.

Chapter 11, Page 239 Lines 16–37: The constant "w" should be "g". "w" denotes George Washington's eldest son, not George Washington. Six occurrences.

Chapter 12, Pages 262–266 In Proposition 12.5.1, Definition 12.6.3 and Definition 12.6.4 it is not necessary to assume that y does not occur in t.

Chapter 12, Page 273 In Definition 12.7.1 interchange the first occurrences of ϕ and ψ.

The following are due to Luke Hunsberger.

Chapter 9, Page 198 In Definition 9.4.6, the reference to Definition 4.6.7 should be replaced with a reference to Definition 4.7.8.

Chapter 9, Page 198 Again in Definition 9.4.6, the added clause should be number 8, not number 9.

The following are due to Dr. Peter Steinacker, Marko Mahling, and Robert Mößgen, at the time (2002) a Professor and two students at the Institute of Logic and Theory of Sciences, University of Leipzig.

Chapter 2, Page 59 The proof of Theorem 2.5.4 is too complicated. It should be done by proving the contraposition, if X is not **K** valid, X has no proof using the **K**-rules. If X is not **K**-valid, there is a world Γ in a model at which X is not true. That means, corresponding to Definition 2.5.1, that the open tableau consisting of a single node, labeled $1\,\neg X$, is satisfiable. By Proposition 2.5.3, it follows that the result of any application of branch extension rules is another satisfiable tableau. Proposition 2.5.2 proves that if a tableau is satisfiable, it is not closed. If every extension of the tableau by branch extension rules yields another satisfiable tableau, no extension yields a closed tableau. Hence $\neg X$ has no tableau proof.

Chapter 5, Page 126 In item 6 of the systematic tableau construction for the existential case, the choice of the parameter has to be restricted to those parameters which don't occur on the branch.

Page 129 and 154 The prefix mapping $\theta(\sigma) = \sigma$ in the model construction seems to be useless and could be omitted.

Chapter 7, Page 153 In the section about completeness it is claimed that the systematic tableau construction procedure *generally* yields an infinite tableau for unprovable sentences. But the resulting tableau is infinite only if the construction rule for the universal quantifier can be applied. Otherwise, e.g. in trying to prove $\neg(\exists x)F(x)$, there will be reached a point where nothing can be added to the tableau, though the construction procedure runs through infinitely many stages.

Chapter 9, Page 192 In the discussion of examples (9.7) and (9.8) you seem to assume that "a nonexistent being has no properties," which contradicts the principles of your varying domain semantics stated e.g. on page 176, that ascribe properties also to nonexistent objects. Or do you only refer to Russell's opinion on the subject? If so, then a remark would be helpful, that the phenomenon called here "a nonexistent being" is handled in your own account by non-designating terms and must not be confused with nonexistent objects.

Chapter 10, Page 214 In the antecedent of the formula A_3 the first of the three occurrences of the individual constant "c" has to be "x".

Chapter 10, Page 214 You are claiming it follows from Proposition 10.2.4 that every instance of item 3 of this Proposition is implied by one or more instances of item 4 and vice versa, i.e. every instance of item 3 is a consequence of one or more instances of item 4 as local assumptions. But Proposition 10.2.4 only states: If for all $\mathcal{I}_0$ compatible models based on $\mathcal{F}$ all instances of item 3 are valid, then for all $\mathcal{I}_0$ compatible models based on $\mathcal{F}$ all instances of item 4 are valid. From this the claim above doesn't follow. The proof of 10.2.4 doesn't even work for the claim that every instance of item 3 is a consequence of one or more instances of item 4 as global assumptions, since in this case one can not choose the interpretation of the predicates arbitrarily, as you did in order to show that items 3 and 4 imply item 1. The validity of the formulas $A_3 \supset A_4$ and $B_4 \supset B_3$ on page 214 thus brings essentially new and important information, which should be pointed out clearly.

Chapter 10, Page 215 In the second part of the proof of Proposition 10.2.5 you assume that the validity of all instances of item 3 (or respectively of item 4) of Proposition 10.2.4 *in a concrete model* does imply local rigidity of "c" in this model, which isn't shown in 10.2.4. For this it has to be shown that the two arbitrarily chosen predicates $\mathcal{I}(P, \Gamma) = \{\mathcal{I}_0(c, \Gamma)\}$ and $\mathcal{I}(P, \Delta) = \{\mathcal{I}_0(c, \Delta)\}$ in the proof of Proposition 10.2.4 can be defined in every model e.g. by taking for $\Phi(x)$ in the first case $\langle \lambda z.x = z \rangle(c)$ and in the second, $x = y$. It would be easier to show directly by a semantical argument that the validity of the formula $\langle \lambda y.\Box \langle \lambda x.x = y \rangle(c) \rangle(c)$ in a model implies local rigidity of "c" in that model.

Chapter 11, Page 234 $(v * l)(t)$ is not a correct expression, because every interpretation is related to a world Δ. It should better be written as $(v * l)(t, \Delta)$, where Δ is a possible world in the model.

Chapter 11, Page 245 In Definition 11.5.1 "if it" should replace "it if".

Chapter 11, Page 254 It would have helped the understanding of Definition 12.3.1 if there had been a remark that the x-variants are not restricted to the actual world.

Chapter 12, Page 266 In the formula of Definition 12.6.4 after the first occurrence of the sign for the material implication, instead of "y" you should allow arbitrary parameters and grounded terms to occur there, since except for parameters, no other free variables are allowed to occur in tableau proofs (as you laid down on page 117).

This correction is due to Jack Woods.

Chapter 2, Page 52 In the table on the middle of the page, the entry for **B** should list B, T, instead of B, 4.

The next correction is due to both William Hanson and Jack Woods.

Proposition 4.8.2, Page 106 This proposition is false as it stands. For a counterexample, take Φ to be $(\exists x)(F(x) \vee \neg F(x))$. Then Part II of the proof (p. 107) cannot be completed, because this instance of the claim is false. The problem can be fixed by changing Prop. 4.8.2 so that it ends:

...if and only if Φ^{E} is valid in every constant domain model in which the extension of the predicate E is nonempty at each world.

Part II can now be proved straightforwardly, and the proof of Part I need not be changed at all, except perhaps to point out that the constant domain model constructed there already satisfies the added condition.

Exercise 8.10.1, Page 185 Mark Alfano pointed out that this problem can not be done as stated. The problem should read: Give a varying domain **B** proof of $\Box CB \supset B$.

Exercise 8.10.2, Page 186 The logic used should not be **K** as stated, but should be **B**.

The following corrections are due to Dilip Ninan, at Tufts.

Proposition 4.6.9, Page 98 While this is correct as stated, it is not sufficient for its intended applications in Chapter Five. As Prof. Ninan points out, the universal quantifier case for the Key Fact, pages 129–130, breaks down because $\Psi(x)$ might contain free occurrences of p_σ, making the application of Proposition 4.6.9 impossible. Similarly for Exercise 5.3.1. Here is Prof. Ninan's restatement of Proposition 4.6.9, which corrects the problem.

Suppose $\mathcal{M} = \langle \mathcal{G}, \mathcal{R}, \mathcal{D}, \mathcal{I} \rangle$ is a constant [or varying] domain model, $\Gamma \in \mathcal{G}$, and v_1 and v_2 are valuations in $\mathcal{M}$. Suppose $\Phi(x)$ is a formula which may have some free occurrences of x **but in which** y **has no bound occurrences**, and $\Phi(y)$ is the result of replacing all free occurrences of x with occurrences of y. And suppose v_1 and v_2 agree on all free variables of $\Phi(x)$ with the possible exception of x, and $v_1(x) = v_2(y)$. Then:

$$\mathcal{M}, \Gamma \Vdash_{v_1} \Phi(x) \text{ iff } \mathcal{M}, \Gamma \Vdash_{v_2} \Phi(y)$$

Exercise 8.2.2, Page 166 The reader is asked to show the validity in all varying domain models of the following.

$$[(\forall x)\Phi(x) \wedge E(x)] \supset \Phi(z)$$

A condition that z is substitutable for x in $\Phi(x)$ must be added, or the formula is not valid. Specifically, we must require that in $\Phi(x)$, no free occurrence of x is within the scope of a quantifier whose variable is z.