

Corrections for Incompleteness in the Land of Sets

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The following corrections are due to Arnon Avron.

1. P. 9, Prop. 5.5, item 4: replace ‘ x ’ by ‘ s ’.
2. P. 23, item 1 (line 10 from the bottom): it is necessary to add the condition that x should not occur free in t .
3. Definition 2.1 (P. 24): again, it is necessary to add the condition that x should not occur free in t . The same applies to Definition 1.1 on P. 39.
4. P. 25, Definition 2.3, item 3: x and y should be *different* variables. The same applies to Definition 1.3 on P. 40.
5. The last paragraph of P. 27 is rather unclear. Definition 2.3 relies on Definition 2.2, where no function or constant symbols are allowed in a Δ_0 formula. So what is the new definition of a Δ_0 formula when function and constant symbols are allowed? Is it “a formula in which all quantifiers are bounded”? If so, then this should be explained, and the constraint noted in (3) above should be emphasized. Moreover: I see no point in allowing the use of “ $(\forall x \in t)\varphi(x)$ ” with complex t in Δ_0 formulas, but insisting on t being a variable in the passage from Δ_0 formulas to Σ formulas. So I suggest to define the notion of a “ Σ formula in an extended sense” in precise terms, and then prove that every such formula is equivalent to a Σ formula in the strict sense.

I suggest also to be more cautious in the process described on P. 28, so it will be clear to the students that (for example) $(\forall x \in A(x, x))\varphi$ is *not* equivalent to a Σ formula, (and why the process described on P. 28 fails for formulas of this type).
6. P. 37, Exercise 8.6: the hint refers to Exercise 5.3 *of Chapter 1*.
7. P. 44, the proof of Theorem 3.2: the variable j is overloaded, and this causes confusion. Note also that on line 9, $n!$ should be $j!$, and on line 13 k_1 should be k_i .
8. P. 54, Exercise 4.1: the hint refers to Exercise 2.3 *of Chapter 2*.
9. P. 54, line 9 from the bottom: replace “ $g_k is \emptyset$ ” by “ $g_k is \emptyset$ ”
10. P. 61, the proof of Theorem 9.2 (which comes before the theorem): as far as I can see, the present proof is correct only if the term that names A is uniquely determined (or else some extra assumptions are imposed on φ). This is not the case according to the current Definition 9.1.

11. P. 69, line 5 from the bottom: “looses” should be “loses”
12. P. 69, last three lines: “sentence” should be “formulas”
13. P. 71, Definition 3.1:
 - (a) At the last line of the page, add \neg before $\varphi(t_i)$.
 - (b) I think that for the argument in the next page one more clause (7) is needed: that if $\neg\neg\varphi$ is in S then so is φ .
14. P. 72, the paragraph before Theorem 3.3: I believe that a hint concerning the cases of atomic formulas and their negations will be helpful for most readers.
15. P. 106, Proof of Corollary 5.6: add “consistent” to the list of properties of TS (in the first sentence).

The next correction is due to Matteo Plebani,
from the University of Santiago de Compostela, Spain.

1. P. 26. A Δ_0 formula, $(\forall y \varepsilon x)\neg(y \varepsilon x)$ is given as something to represent the empty set. In fact, it represents $\{\emptyset\}$ and not \emptyset . Following Plebani’s suggestion, use $(\exists y \varepsilon x)\neg(y \varepsilon x)$ instead.