Corrections for

*Possible world semantics for the first-order logic of proofs*

Annals of Pure and Applied Logic 165 (2014) 225-240

and also

*Possible world semantics for first-order $\mathsf{LP}$*

CUNY Ph.D. Program in Computer Science technical report

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Meghdad Ghari wrote "I have difficulty proving the validity of the proof checker axiom in your proposed semantics," and supplied technical details. His objection was correct, and I propose the following amendments to the Annals paper. Similar modifications apply to my Technical Report, but I do not state them explicitly.

The problem is in the ! Condition of Definition 3.5 of the paper. As stated it reads: "$\mathcal{E}(t, A) \subseteq \mathcal{E}(!t, t{:}_X A)$ where $X$ is the set of domain constants in $A$." This should be revised to the following. "If $\Gamma \in \mathcal{E}(t, A)$, $X \subseteq \mathcal{D}(\Gamma)$, and $X$ contains all domain constants in $A$, then $\Gamma \in \mathcal{E}(!t, t{:}_X A)$."

A similar change is needed in Definition 11.1 too.

In Section 5, Soundness, an argument is made for condition B4, the validity of $t{:}_X A \rightarrow !t{:}_X t{:}_X A$. This is actually shown for a representative special case, but the case chosen is not fully representative. As given, validity is shown for $X = \{x\}$ and $A = A(x, y)$. However, it is allowed that $X$ contain variables not free in $A$ and this possibility is missing in the special case used in the soundness proof. Suppose we consider the same $A$, but $X = \{x, z\}$ instead. That is, we must show the validity of $t{:}_{\{x,z\}} A(x, y) \rightarrow !t{:}_{\{x,z\}} t{:}_{\{x,z\}} A(x, y)$.

Let $\Gamma \in \mathcal{G}$ and consider the $\mathcal{D}(\Gamma)$ instantiation resulting from the substitution $\{x/a, z/b\}$ where $a, b \in \mathcal{D}(\Gamma)$. We will show $\mathcal{M}, \Gamma \Vdash t{:}_{\{a,b\}} A(a, y) \rightarrow !t{:}_{\{a,b\}} t{:}_{\{a,b\}} A(a, y)$. Suppose $\mathcal{M}, \Gamma \Vdash t{:}_{\{a,b\}} A(a, y)$.

First, $\Gamma \in \mathcal{E}(t, A(a, y))$ so by the *revised* !-Condition of Definition 3.5, $\Gamma \in \mathcal{E}(!t, t{:}_{\{a,b\}} A(a, y))$. (This failed under the original !-Condition).

Next, suppose $\Gamma \mathcal{R} \Delta$ and $\Delta \mathcal{R} \Omega$. Since $\mathcal{R}$ is transitive, $\Gamma \mathcal{R} \Omega$ and since $\mathcal{M}, \Gamma \Vdash t{:}_{\{a,b\}} A(a, y)$ then $\mathcal{M}, \Omega \Vdash A(a, y)$ for every instance of $y$ from $\mathcal{D}(\Omega)$. Also since $\Gamma \in \mathcal{E}(t, A(a, y))$ then $\Delta \in \mathcal{E}(t, A(a, y))$ by the $\mathcal{R}$ Closure Condition of Definition 3.5. Since $\Omega$ is arbitrary, $\mathcal{M}, \Delta \Vdash t{:}_{\{a,b\}} A(a, y)$. And since $\Delta$ is arbitrary, $\mathcal{M}, \Gamma \Vdash !t{:}_{\{a,b\}} t{:}_{\{a,b\}} A(a, y)$.

In Section 8 canonical models are constructed. There is no change in the definition, but it must be shown that the canonical model meets the revised condition for $\mathcal{E}$. Here is the argument.

Suppose $\mathcal{M} = \langle \mathcal{G}, \mathcal{R}, \mathcal{D}, \mathcal{I}, \mathcal{E} \rangle$ is a canonical model, Definition 8.1. Assume $\Gamma \in \mathcal{G}$, $\Gamma \in \mathcal{E}(t, A)$, $X \subseteq \mathcal{D}(\Gamma)$, and $X$ contains all domain constants in $A$. We show $\Gamma \in \mathcal{E}(!t, t{:}_X A)$. For the argument, let $Y$ be exactly the set of domain constants in $A$, and so $Y \subseteq X \subseteq \mathcal{D}(\Gamma)$.

Since $\Gamma \in \mathcal{E}(t, A)$, by definition $t{:}_Y A \in \mathsf{form}(\Gamma)$. Since $\mathsf{form}(\Gamma)$ is maximally consistent, repeated use of axiom **A3** yields that $t{:}_X A \in \mathsf{form}(\Gamma)$. And then axiom **B4** gives us that $!t{:}_X t{:}_X A \in \mathsf{form}(\Gamma)$. Note that the set of witness variables in $t{:}_X A$ is exactly $X$. It follows from the definition of $\mathcal{E}$ in the canonical model that $\Gamma \in \mathcal{E}(!t, t{:}_X A)$.

Meghdad Ghari also reports the following typos in the *Annals* paper.

1. (Page 225) In the third sentence of the Abstract: "the tech report proved an arithmetic completeness theorem" should be "the tech report proved an arithmetic soundness theorem". Indeed, Corollary 6 of the tech report of Artemov and Yavorskaya shows that completeness is not attainable.

2. (Page 234, line 6 from bottom) The last sentence of Definition 7.2 reads "If $c{:}_{\emptyset}A \in \mathcal{C}$, put $c{:}_{\emptyset}A' \in \mathcal{C}(W)$." It should be "If $c{:}_{\emptyset}A' \in \mathcal{C}$, put $c{:}_{\emptyset}A \in \mathcal{C}(W)$".

3. (Page 236, line 3) In the **Specification of $\mathcal{G}$**, item 3 ends with "constant specification $\mathcal{C}$" but it should be "constant specification $\mathcal{C}(\mathsf{var}(\Gamma))$".

4. (Page 239, line 12 from bottom.) The sentence immediately following Definition 11.1 contains "closed formula $A$ of language $L(D)$." This should be "closed $D$-formula $A$". It has nothing to do with Definition 7.1.