

# Exercises

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October 20, 2015

1. Provide the details of a soundness proof for prefixed tableaux.
2. Prove that a  $K$  downward saturated set is  $K$  satisfiable.
3. Give prefixed tableau proofs for the following.
  - (a) In  $T$ ,  $\diamond(P \supset \Box P)$
  - (b) In  $K4$ ,  $(\Box X \wedge \Box Y) \supset \Box(\Box X \wedge \Box Y)$
  - (c) In  $S4$ ,  $(\Box \diamond X \wedge \Box \diamond Y) \supset \Box \diamond(\Box \diamond X \wedge \Box \diamond Y)$ . Can it be proved in  $K4$ ?
  - (d) In  $S5$ ,  $\Box(\diamond P \supset P) \supset \Box(\diamond \neg P \supset \neg P)$
  - (e) In  $S5$ ,  $\diamond(P \wedge \Box Q) \equiv (\diamond P \wedge \Box Q)$
4. Give nested sequent proofs for (3a) and (3b) above. You might convert the prefixed tableau proofs you (should) have constructed.
5. Give destructive tableau proofs for (3a) and (3b) above.
6. The following is not provable in  $K$ .  $\Box(X \wedge \Box Y) \supset \Box(\Box X \wedge Y)$ . Construct a (failed) systematic, fair proof attempt using the prefixed tableau rules. Extract a counter-model from the result.