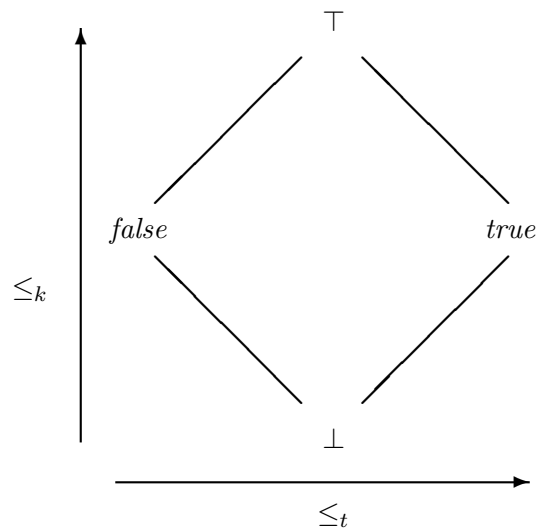


Exercise Set 10

Melvin Fitting

April 17, 2018

Recall, *first degree entailment*, FDE, is the four-valued logic whose operations \wedge and \vee are the meet and join operations under the truth ordering in the following lattice structure, and whose \neg operation switches *true* and *false* but leaves \perp and \top unchanged.



Also recall that the *designated* truth values are *true* and \top . Let \mathcal{D} be the designated set, $\{true, \top\}$. A valuation maps propositional variables to the four truth values of FDE. A valuation extends to all formulas using the operations \wedge , \vee , and \neg just described (we assume we don't have an implication now). Finally, recall that $S \vdash_{\text{FDE}} X$ means that every valuation that maps each formula in S to a value in \mathcal{D} also maps X to a member of \mathcal{D} .

Let A and B be formulas. Here are various conditions relating A and B that we want to examine.

- (i) $\{A\} \vdash_{\text{FDE}} B$ and $\{\neg B\} \vdash_{\text{FDE}} \neg A$
- (ii) Any valuation mapping A to *true* maps B to *true*
- (iii) Any valuation mapping B to *false* maps A to *false*
- (iv) Any valuation mapping A to *false* maps B to *false*
- (v) $\{B\} \vdash_{\text{FDE}} A$ and $\{\neg A\} \vdash_{\text{FDE}} \neg B$

(vi) Every valuation gives A and B exactly the same truth value

Here are the homework problems. For the first three, either give a proof or a counter-example.

1. Does (i) imply (ii)
2. Does (i) imply (iii)
3. Does (i) imply (iv)
4. Show (i) and (v) together imply (vi)