Exercise Set 11

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The relevance logic B was characterized semantically in class. All questions here ask for you to use that semantics. To show non-validity, give a counter-model and explain why it is one. To show validity, establish that there can be no counter-mnodel.

1. Show the validity or non-validity in B of $((A \supset B) \wedge (A \supset C)) \supset (A \supset (B \wedge C))$.

2. Show the validity or non-validity in B of $(P \supset Q) \supset ((P \wedge R) \supset (Q \wedge R))$.

3. This question amounts to a proof that B is a relevance logic. That is, if $A \supset B$ is valid in B then A and B must share a propositional variable. (It is taken directly from Graham Priest's book)

   (a) Let $\perp$ and $\perp^*$ be a pair of non-normal worlds such that every propositional parameter is true at $\perp$ and false at $\perp^*$. Suppose that $R\perp\perp\perp$, $R\perp^*\perp\perp^*$, and that each world accesses no other worlds. Show that every formula is true at $\perp$ and false at $\perp^*$.

   (b) Let w and w^* be a pair of non-normal worlds such that $Rw\perp w$ and $Rw^*\perp w^*$. Using part (a), show that: (i) if every parameter in A is true at w and false at w^*, the same is true of A; (ii) if every parameter in B is false at w and true at w^*, the same is true of B.

   (c) Use this to show that if $A \supset C$ is valid in B, then A and C share a propositional parameter.