Exercise Set 5

Melvin Fitting

March 6, 2018

1. In the modal logic K, two of the following are provable, one is not. Give proofs for those that are, and a counter-model for the one that is not.
   (a) $\Box(P \supset \Box\Box R) \supset \Box(P \supset \Box(\Diamond Q \supset \Diamond R)$
   (b) $\Box(P \supset \Box\Box R) \supset \Box(P \supset \Box(Q \supset \Diamond R))$
   (c) $\Box\Box(Q \supset \Diamond R) \supset \Box(P \supset \Box(Q \supset \neg\Box\neg R))$
2. Using the previous question, is the formula that is not provable in K provable in: T, $\mathsf{S4}$, $\mathsf{S5}$?
3. This is about modal deduction or consequence, in the logic K. One of the following has a proof. Show a closed tableau for this. (Destructive or Priest style). One does not have a proof. Give a counter-model for it.
   (a) $\emptyset \models_{\mathsf{K}} \{\Box A \supset \Box\Box A\} \longrightarrow \Box A \supset \Box\Box\Box A$
   (b) $\{\Box A \supset \Box\Box A\} \models_{\mathsf{K}} \emptyset \longrightarrow \Box A \supset \Box\Box\Box A$