Exercise Set 7

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Consider the following propositional axiom system.

Axiom Schemes

1. $(P \supset (Q \supset R)) \supset ((P \supset Q) \supset (P \supset R))$
2. $P \supset (Q \supset P)$
3. $P \supset (P \vee Q)$
4. $Q \supset (P \vee Q)$
5. $(P \supset R) \supset ((Q \supset R) \supset ((P \vee Q) \supset R)$

Rule of Inference $\dfrac{X \quad X \supset Y}{Y}$

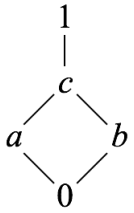
The problem is to show that $(P \supset Q) \vee (Q \supset P)$ is *not* provable. We use the following matrices.

$\supset$	0	a	b	c	1
0	1	1	1	1	1
a	b	1	b	1	1
b	a	a	1	1	1
c	0	a	b	1	1
1	0	a	b	c	1

$\vee$	0	a	b	c	1
0	0	a	b	c	1
a	a	a	c	c	1
b	b	c	b	c	1
c	c	c	c	c	1
1	1	1	1	1	1

Designated value is only 1.

These matrices derive from the following partial order.



1. Show that axioms 2 and 3 evaluate to 1 under all truth value assignments.
2. Show that always evaluating to 1 is preserved under *modus ponens*.
3. Finally, find values for which $(P \supset Q) \vee (Q \supset P)$ is not provable.