Strict Implication

From *Symbolic Logic* by Clarence Irving Lewis and Cooper Harold Langford, 1932/1959. Primitive connectives, $\sim$, $\wedge$, and $\Diamond$. Disjunction is defined as usual, and strict implication is defined as $(P \prec Q)$ abbreviates $\sim \Diamond(P \wedge \sim Q)$. Also define strict equivalence by $P = Q$ abbreviates $(P \prec Q) \wedge (Q \prec P)$.

Rules:

Substitution Of Strict Equivalents If $P = Q$ has been proved, replacing P in an expression with Q produces an equivalent expression.

Adjunction From P and Q conclude $P \wedge Q$

Strict Inference From P and $P \prec Q$ conclude Q

Axioms:

B–1 $(P \wedge Q) \prec (Q \wedge P)$

B–2 $(P \wedge Q) \prec P$

B–3 $P \prec (P \wedge P)$

B–4 $((P \wedge Q) \wedge R) \prec (P \wedge (Q \wedge R))$

B–5 $P \prec \sim\sim P$

B–6 $((P \prec Q) \wedge (Q \prec R)) \prec (P \prec R)$

B–7 $(P \wedge (P \prec Q)) \prec Q$

This much gives system S1. B–5 turned out to be redundant.

Adding the following "consistency postulate" gives S2. $\Diamond(P \wedge Q) \prec \Diamond P$

S3 adds to S1 (not to S2) the following axiom. $((P \prec Q) \supset (\neg\Diamond Q \prec \neg\Diamond P))$

S4 is the set B–1 through B–7, together with $\Diamond\Diamond P \prec \Diamond P$ It includes S2.

Material implication can be introduced by setting $P \supset Q$ to mean $\sim (P \wedge \sim Q)$. It will have its classical properties, but it is different than strict implication.