equivalence to Beth tableau systems (using cut elimination) then proving completeness for these by a systematic tableau style construction. This was complex and hard to follow, and soon Henkin-style completeness arguments became standard. A tree-style system for S4 appeared in [24], similar to the intuitionistic system of that book. Systems for several modal logics, based on somewhat different principles, appeared in [25], and for temporal logics in [73]. But the most extensive development was in Fitting's 1983 book [26] which, among other things, gave tableau systems for dozens of normal and non-normal modal systems. We sketch a few to give an idea of the style of treatment, and the intuition behind the tableaus.

3.3.1 Destructive Tableau Systems

Fitting extended Smullyan's uniform notation to the modal case. Here is a signed-formula version. The idea is: a ν formula is true at a possible world if and only if the corresponding ν_0 is true at every accessible world; a π formula is true at a possible world if the corresponding π_0 is true at some accessible world.

ν	ν_0
$T\,\Box X$	$T\,X$
$F\,\Diamond X$	$F\,X$

π	π_0
$F\,\Diamond X$	$F\,X$
$T\,\Box X$	$T\,X$

Next, if S is a set of signed formulas, a set $S^{\#}$ is defined. The idea is, if the members of S are true at a possible world, and we move to a 'generic' accessible world, the members of $S^{\#}$ should be true there. The definition of $S^{\#}$ differs from modal logic to modal logic. We give the version for K, the smallest normal modal logic.

$$S^{\#} = \{\nu_0 \mid \nu \in S\}$$

The rules for K are exactly as in the classical Smullyan system, together with the following 'destructive' rule.

$$\frac{S, \pi}{S^{\#}, \pi_0}$$

Unlike the other rules (but exactly like the intuitionistic rules in section 3.1), this one modifies a whole branch. If S, π is the set of signed formulas on a branch, the whole branch can be replaced with $S^{\#}, \pi_0$. Since this removes formulas, and modifies others, it is an information-loosing rule, hence the description 'destructive.' We continue to use the device of checking off deleted formulas in trees.

Here is a simple example of a proof in this system, of $\Box(X \supset Y) \supset (\Box X \supset \Box Y)$. It begins as follows.

$$
\begin{array}{lll}
\surd & 1. & F\,\Box(X \supset Y) \supset (\Box X \supset \Box Y) \\
\surd & 2. & T\,\Box(X \supset Y) \\
\surd & 3. & F\,\Box X \supset \Box Y \\
\surd & 4. & T\,\Box X \\
\surd & 5. & F\,\Box Y \\
 & 6. & T\,X \supset Y \\
 & 7. & T\,X \\
 & 8. & F\,Y
\end{array}
$$

Here 2 and 3 are from 1, and 4 and 5 are from 3 by $F \supset$. Now take 5 as π, and 1 through 4 as S, and apply the modal rule. Formula 1 is simply deleted; 2 is deleted but 6 is added; 3 is deleted; 4 is deleted but 7 is added (at this point, S has been replaced by $S^{\#}$); and finally 5 is deleted but 8 is added (this is π_0). Now an application of $T \supset$ to 6 produces a closed tableau.

The underlying intuition is direct. All rules, except the modal one, are seen as exploring truth at a single world. The modal rule corresponds to a move from a world to an alternative one. A soundness argument can easily be based on this. Completeness can be proved using either a systematic tableau construction or a maximal consistent set approach. As is the case with both classical and intuitionistic tableaus, interpolation theorems and related results can be derived from the tableau formulation.

Several other normal modal logics can be treated by modifying the definition of $S^{\#}$, or by adding rules, or both. For instance, the logic $K4$ (adding transitivity to the model conditions) just requires a change in a definition, to the following.

$$S^{\#} = \{\nu, \nu_0 \mid \nu \in S\}$$

Generally speaking, modal logics that have tableau systems of this kind can not have a semantics whose models involve *symmetry* of the accessibility relation. Interestingly enough, though, such logics can often be given tableau systems in this style if a cut rule is allowed, and in fact a *semi-analytic* version is enough. Semi-analyticity extends and weakens the notion of analytic cut, but is still not as broad as the unrestricted version. See Fitting, [26], for more details.

Various regular but non-normal logics can be dealt with by restricting rule applicability (see [26, 30] for a definition of regularity). For instance, if we use the system for K above, but restrict the modal rule to those cases in which $S^{\#}$ is non-empty, we get a tableau system for the smallest regular logic C. It is even possible to treat such quasi-regular logics as $S2$ and $S3$ by similar techniques.