

Notes on Three Valued Logics

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We are beginning a discussion of many valued logics with the simplest examples, *three valued logics*. Even the addition of one more truth value to the classical ones can be done in quite a few ways, and of these many have good and interesting motivations. We will cover the motivations in class. What is here is just the truth tables, and some consequences. For each logic I will use **t** and **f** for the ‘classical’ truth values (Priest uses 1 and 0), and **n** for the third truth value. Sometimes a difference choice of notation is often used, and I will say so as appropriate.

1 Strong Kleene, K_3

\wedge	t	n	f	\vee	t	n	f	\supset	t	n	f	\neg	
	t	n	f		t	t	t		t	t	n	f	f
	n	n	n		n	t	n		n	t	n	n	n
	f	f	f		f	t	n		f	t	t	f	t

Designated truth values: $\{\mathbf{t}\}$. Commonly used for **n**: \perp , **u**.

2 Weak Kleene

\wedge	t	n	f	\vee	t	n	f	\supset	t	n	f	\neg	
	t	t	n		t	t	n		t	t	n	f	f
	n	n	n		n	n	n		n	n	n	n	n
	f	f	n		f	t	n		f	t	n	f	t

Designated truth values: $\{\mathbf{t}\}$. Commonly used for **n**: \perp , **u**.

3 Asymmetric Logic

\wedge	t	n	f	\vee	t	n	f	\supset	t	n	f	\neg	
	t	t	n		t	t	t		t	t	n	f	f
	n	n	n		n	n	n		n	n	n	n	n
	f	f	f		f	t	n		f	t	t	f	t

Designated truth values: $\{\mathbf{t}\}$. Commonly used for **n**: \perp , **u**.

4 Łukasiewicz Logic, L_3

The same as Strong Kleene, except for implication.

\wedge	t	n	f	\vee	t	n	f	\supset	t	n	f	\neg	
t	t	n	f	t	t	t	t	t	t	n	f	t	f
n	n	n	f	n	t	n	n	n	t	t	n	n	n
f	f	f	f	f	t	n	f	f	t	t	t	f	t

Designated truth values: $\{\mathbf{t}\}$. Commonly used for **n**: \perp , **u**.

5 Logic of Paradox, *LP*

The tables are exactly the same as for Strong Kleene, but the designated values are not.

\wedge	t	n	f	\vee	t	n	f	\supset	t	n	f	\neg	
t	t	n	f	t	t	t	t	t	t	n	f	t	f
n	n	n	f	n	t	n	n	n	t	n	n	n	n
f	f	f	f	f	t	n	f	f	t	t	t	f	t

Designated truth values: $\{\mathbf{t}, \mathbf{n}\}$. Sometimes used for **n**: \top , **u**.

6 *RM*₃

The same as *LP*, except for implication.

\wedge	t	n	f	\vee	t	n	f	\supset	t	n	f	\neg	
t	t	n	f	t	t	t	t	t	t	f	f	t	f
n	n	n	f	n	t	n	n	n	t	n	f	n	n
f	f	f	f	f	t	n	f	f	t	t	t	f	t

Designated truth values: $\{\mathbf{t}, \mathbf{n}\}$. Sometimes used for **n**: \top , **u**.