# Modal Logic

## Sample Questions

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1. Prove the following using the $\mathsf{S5}$ tableau rules:

$$\Diamond(P \wedge \Diamond(Q \wedge \Box R)) \supset (\Diamond(P \wedge R) \wedge \Diamond(Q \wedge R))$$

   This is one example. More questions like this are on page 51 and 55.

2. Consider the following formula.

$$[(\exists x)\Diamond P(x) \wedge \Box(\forall x)(P(x) \supset Q(x))] \supset (\exists x)\Diamond Q(x)$$

   For each of first-order *varying domain* $\mathsf{K}$ and *constant domain* $\mathsf{K}$ either give a tableau proof, or give a model showing the formula is not valid.

   More questions like this are on pages 101 and 115.

3. Use varying domain $\mathsf{K}$, under the assumption that terms always designate. Consider the following three formulas:

   (a) $\langle \lambda x. \Box \langle \lambda y. P(y) \rangle (x) \rangle (c)$
   (b) $\langle \lambda x. \langle \lambda y. \Box P(y) \rangle (x) \rangle (c)$
   (c) $\Box \langle \lambda y. P(y) \rangle (c)$

   where $P$ is a one-place relation symbol and $c$ is a constant symbol. Exactly two of these formulas are always equivalent. Use tableaus to show that two of them imply each other. Give a model to show the third neither implies nor is implied by the others.

   More questions like this are on page 200.

4. Give a tableau proof in constant domain $\mathsf{K}$, under the assumption that terms always designate, of the following

$$\langle \lambda y. \Box \langle \lambda x. x = y \rangle (c) \rangle (c) \supset [\langle \lambda x. \Box \varphi(x) \rangle (c) \supset \Box \langle \lambda x. \varphi(x) \rangle (c)]$$

   More questionns like this are on page 226-227.

5. The following are from Chapter Twelve, section 4.

   (a) Let $P$ be a one-place relation symbol. Give a model showing that $\langle \lambda x. P(x) \rangle (\imath x. P(x))$ is not valid.
   (b) Show the validity of the following:

$$\langle \lambda x. \psi(x) \rangle (\imath x. \varphi(x)) \supset \mathsf{D}(\imath x. \varphi(x)).$$

   More questions like this are on page 272-273.