Modal Logic

Sample Questions

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1. Prove the following using the S5 tableau rules:

$$\Diamond(P \wedge \Diamond(Q \wedge \Box R)) \supset (\Diamond(P \wedge R) \wedge \Diamond(Q \wedge R))$$

   More questions like this are in the Exercise set for Chapter 7 Section 2.

2. Consider the following formula.

$$[(\exists x)\Diamond P(x) \wedge \Box(\forall x)(P(x) \supset Q(x))] \supset (\exists x)\Diamond Q(x)$$

   For each of first-order *varying domain* K and *constant domain* K either give a tableau proof, or give a model showing the formula is not valid.

   More questions like this are in Exercises 9.1.1, 9.1.2, 9.2.1.

3. Give a tableau proof in constant domain K, under the assumption that terms always designate, of the following

$$\langle \lambda y.\Box\langle \lambda x.x = y\rangle(c)\rangle(c) \supset [\langle \lambda x.\Box\varphi(x)\rangle(c) \supset \Box\langle \lambda x.\varphi(x)\rangle(c)]$$

   More questions like this are in the Exercise set for Chapter 17 Section 2.

4. For the following the setting is *CN*.

   (a) Give a model showing that $\langle \lambda x.P(x)\rangle(\imath x.P(x))$ is not valid.

   (b) Show the validity of $\langle \lambda x.\psi(x)\rangle(\imath x.\varphi(x)) \supset \mathsf{D}(\imath x.\varphi(x))$. Also give a tableau proof.

   (c) Show the validity of $\mathsf{D}(\imath x.\varphi(x)) \supset \langle \lambda x.\psi(x)\rangle(\imath x.\varphi(x))$. Also give a tableau proof.

   More questions like this are in the Exercise set for Chapter 20 Section 4.